This assignment combines a feedback control problem and your experimental data collected in the lab. You may want to do the numerical analysis before you go into the lab. You will examine the behavior of the standard controller types and compare theoretical calculations with experimental results. You may want to type your responses to the short answer questions.

The lab is worth 10% of your course grade. Perform the experiment in the laboratory in groups of two or three people, but complete the write-up individually.

We will explore the control of the height of a liquid in a tank (h) using P, PI, and PID control. A 91 liter cylindrical tank is 38 cm in diameter. Water flows into the tank at a constant rate of 80 ml/s. The height of the tank is measured with a pressure transducer, and the flow rate of the outlet stream can be controlled with a pneumatic valve according to the relation $q_{out} = \alpha h_{tot}^{1/2}$, where $h_{tot} = h + h_0$ is the head of pressure in tank in cm of water. The coefficient $\alpha = a - bP - cP^2$ [in ml/s-cm$^{1/2}$] , where P is the controller signal.

For (a), use the following parameters:

$$h_0 = 1530 \text{ cm}, a = 2.615 \text{ ml/s-cm}^{1/2}, b = 0.203 \text{ ml/s-cm}^{1/2}, c = 0.022 \text{ ml/s-cm}^{1/2}$$

a.   Show that the (open-loop) transfer function from the control signal P to the height h has a gain of 458 cm and a time constant of 43700 s. If you need to employ a Taylor expansion, work around the operating point $\bar{h} = 20$ cm. Would it be reasonable to represent the transfer function as an integrating process? If so, you may do so for your theoretical calculations.

b.   Draw a block diagram for the closed-loop process. Indicate the variables and units on each connection between blocks, and show the transfer functions inside the blocks.

c.   Derive the closed-loop transfer function, using G_c for the controller. You may combine the measurement and transmission devices into the controller function.

d.   Assume the system is at steady-state with a height of 10 cm. At $t = 0$, the set point is changed to 20 cm.

   • Plot the theoretical response of the system using a P controller for $K_c = 1$, 10, and 100. Overlay your experimental data.
   • Describe the effect of K_c on the process response (use the theoretic calculations).
   • Explain any differences between experiment and theory.

e. Assume the system is at steady-state with a height of 20 cm. At $t = 0$, the set point is changed to 30 cm.

- Plot the response of the system using a PI controller with $K_c = 10$ and $\tau_I = 0.5, 5, 50,$ and 500 s. Overlay your experimental data.
- Describe the effect of τ_I on the process response.
- Explain any differences between experiment and theory.

f. Assume the system is at steady-state with a height of 30 cm. At $t = 0$, the set point is changed to 40 cm.

- Plot the response of the system using a PID controller with $K_c = 10$, $\tau_I = 5s$ and $\tau_D = 1, 10,$ and 100 s.
- Describe the effect of τ_D on the process response.
- Explain any differences between experiment and theory.

g. Plot your four experimental runs together. Describe the responses and explain the effects of each mode of control (P, PI, PID).

h. What parameters did you choose for your fourth run? Why? Did the experimental response match your expectations?

i. What are the sources of error in the experiment? What are sources of error in the theory? What additional effects cause differences between experiment and theory?

j. BONUS: Redo your theoretical plot(s) incorporating whatever factors you have identified which contribute significantly to the difference between experiment and theory. Compare your new plot(s) with the experiment and interpret.